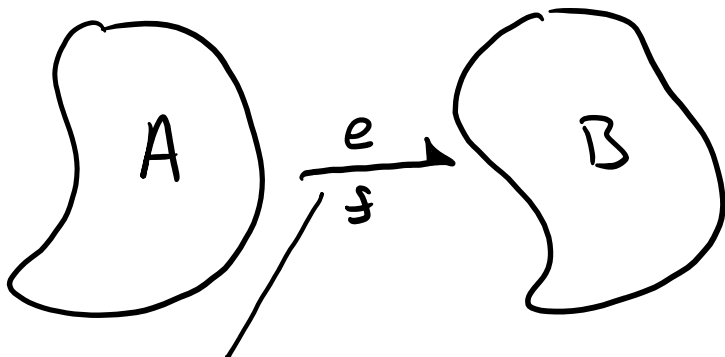


# Bond Graph Basics

System: Entity separable from the universe by a conceptual boundary which made up of interrelated components.  
Energy can cross the boundary to and from the universe and the system can change with time based on this energy exchange.

Model of a system: Simplified abstractions of a real system that changes with time and external energy exchange in an approximation to the real system.

Components A and B make up a system:



bond  
represents the  
power flow  
from A to B  
(Corresponding to  
direction of the  
arrow)

Power is the  
product of:  
effort and  
flow

effort: Voltage, Force  
flow: current, velocity

$$P = e \cdot f$$

Energy exchanged across  
the bond is the time integral

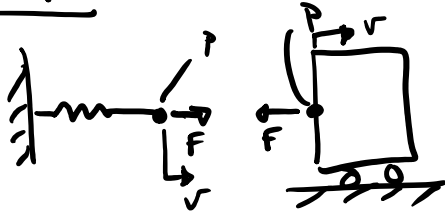
of power:

$$E = \int \mathbb{P} dt = \int e \cdot s dt$$

Effort, flow, and power are functions of time.

$$e(t), f(t), \mathbb{P}(t), E(t)$$

Example: mechanical linear motion



$$F \cdot v = \mathbb{P} \quad \text{power flow at the boundary}$$

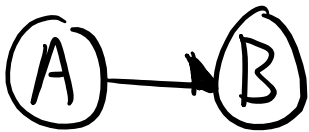
/ \      / \

force velocity

We will use a sign convention (represented by a bond's arrow direction):

$$(\text{positive effort}) \times (\text{positive flow}) = (\text{positive power})$$

i.e.



establishes that a positive quantity of power is from A to B.

Other important variables:

"momentum"  $p = \int e dt$  or  $e = \frac{dp}{dt}$  e.g.  $\Sigma F = \frac{dp}{dt}$

↓  
 $p = mv$

"displacement"  $q = \int f dt$  or  $f = \frac{dq}{dt}$  e.g.  $v = \frac{dx}{dt}$

↓  
 $v = \frac{dx}{dt}$

Effort, flow, momentum, and displacement are the core

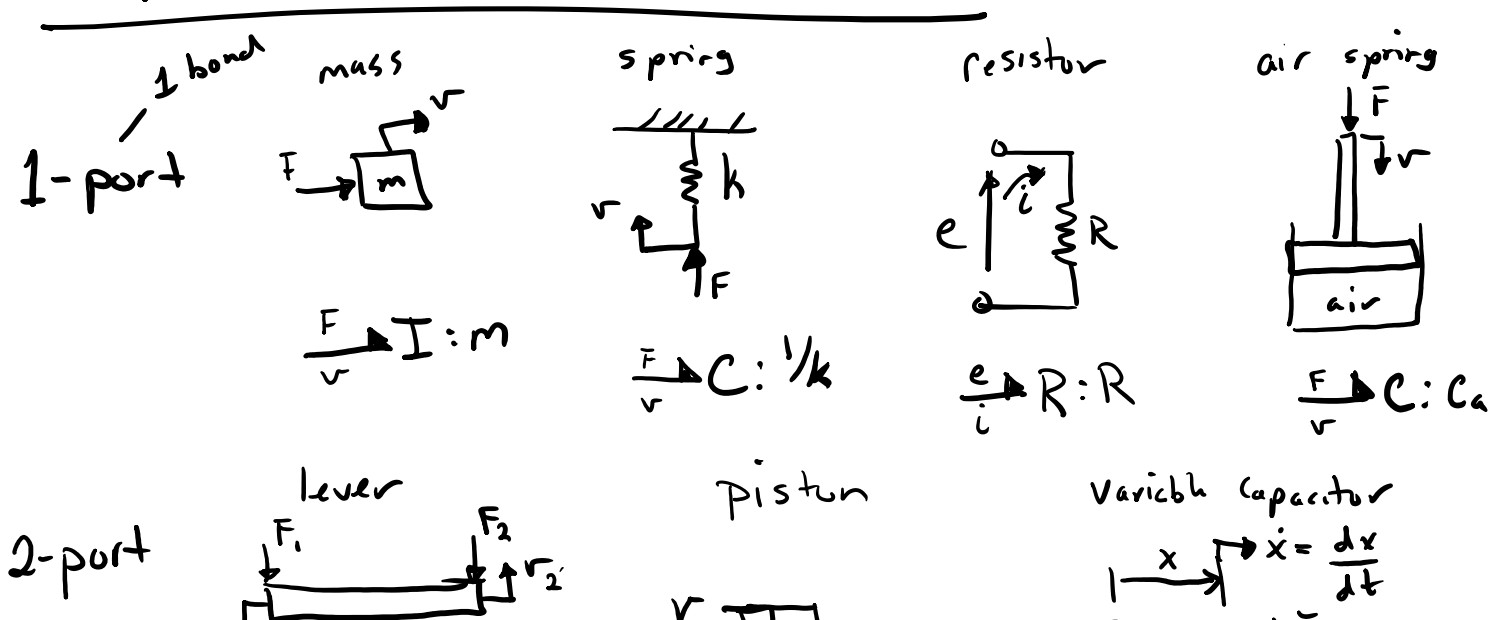
Effort, flow, momentum, and displacement are the core time varying variables to describe a system's change (dynamics).

Four different energy domains:

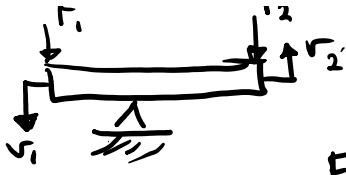
	e	f	P	q
mechanical translation	F [N] force	v [m/s] velocity	P [N·s] linear momentum	x [m] displacement (position)
mechanical rotation	$\tau$ [N·m] torque	$\omega$ [rad/s] angular velocity	h [N·m·s] angular momentum	$\theta$ [rad] angular displacement (angle)
electrical	e [V] voltage	i [A] current	$\lambda$ [V·s] flux	q [A·s] charge <small>Coulomb</small>
hydraulic	P [N/m <sup>2</sup> ] pressure	Q [m <sup>3</sup> /s] flow rate	P <sub>p</sub> [N/m <sup>2</sup> ·s] pressure momentum	V [m <sup>3</sup> ] volume

Stick with core SI fundamental units!

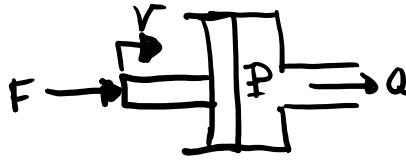
### Examples of systems components



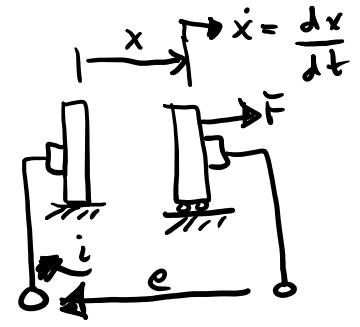
2-port



$$\frac{F_1}{v_1} \rightarrow TF \frac{F_2}{v_2}$$



$$\frac{F}{v} \rightarrow TF \frac{P}{Q}$$



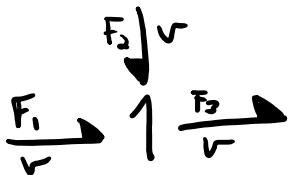
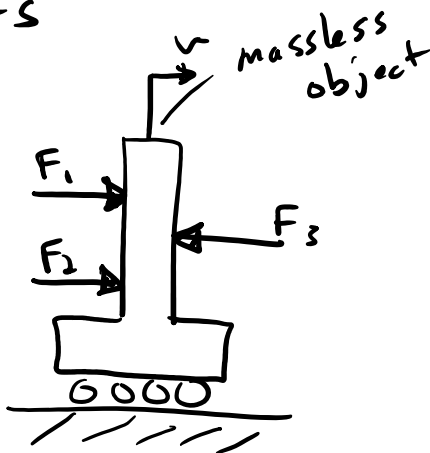
$$\frac{e}{i} \rightarrow TF \frac{F}{\dot{x}}$$

voice coil transducers

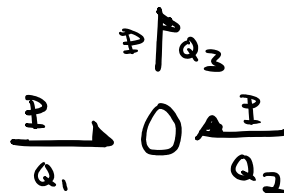
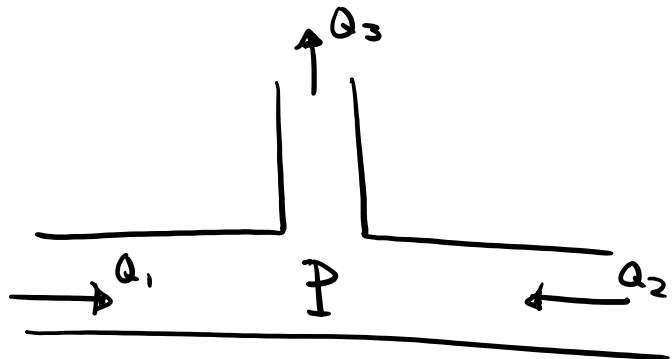


$$\frac{e}{i} \rightarrow GY \frac{F}{v}$$

3 ports



"one junction"  
common flow port/junction

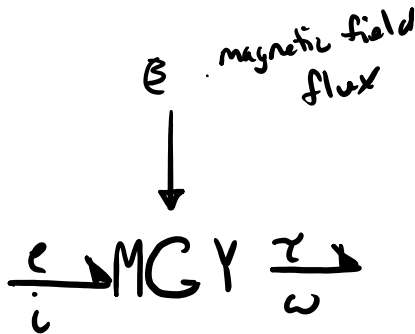
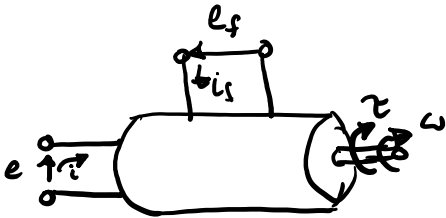


"zero junction"  
common effort junction/port

common flow port/junction  
flows are the same

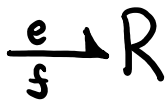
common effort junction/part  
effort is the same

Modulated 2-port



Generalized Form of Basic Components

1-port resistance (relates effort and flow directly)

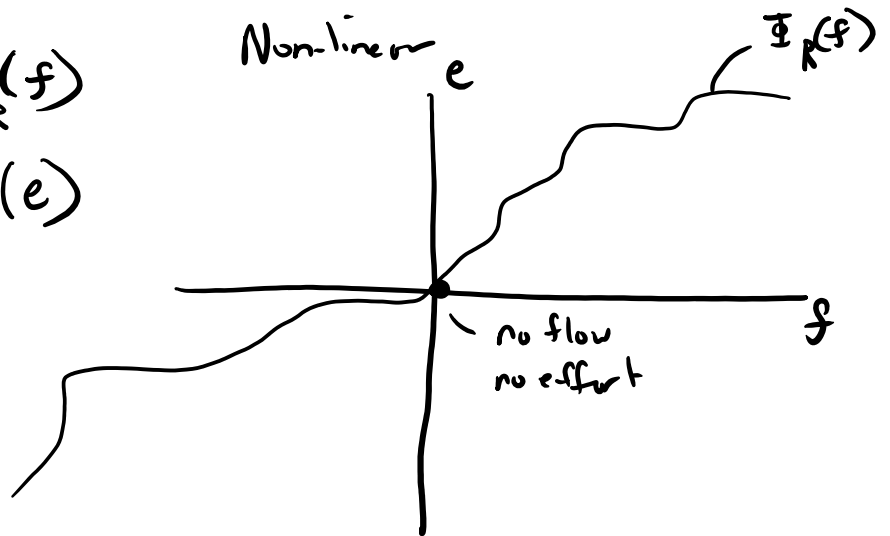


$$e = \Phi_R(f)$$

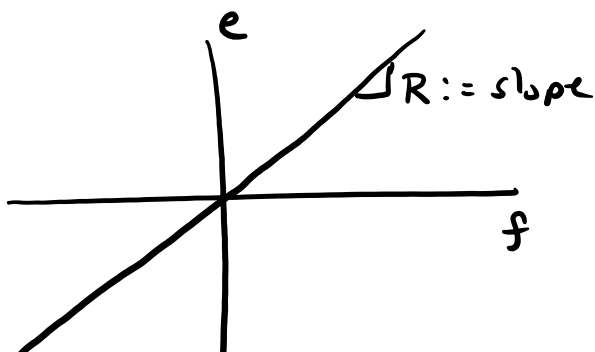
$$f = \Phi_R^{-1}(e)$$

always draw half arrow towards the 1-port! (sign convention)

energy non-conservative part



Linear resistor



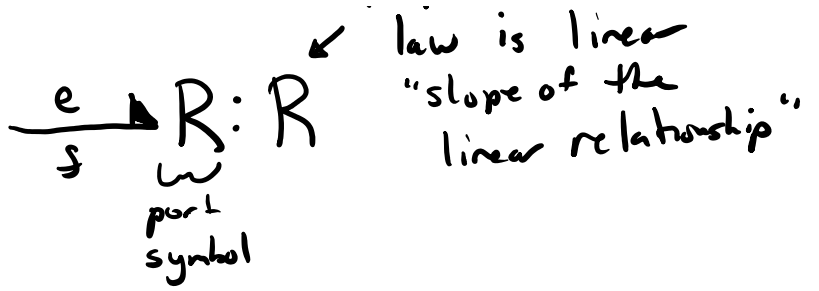
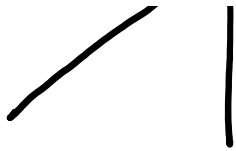
resistance

$$e = R f$$

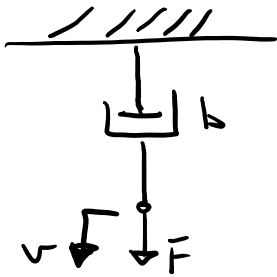
$$f = \frac{1}{R} e$$

conductance

if present, constitutive law is linear "slope of the ...



Common resistances



Viscous linear damper / dashpot

$$\frac{F}{v} \rightarrow R: b$$

$$F = bv$$