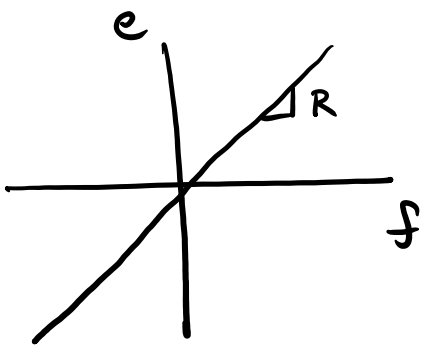


- HW #2 Due Monday
- Lab #1 Due Friday, Jan 24th
- Attend two of three Friday labs for first lab.

Resistance (linear version)



$$e = Rf$$

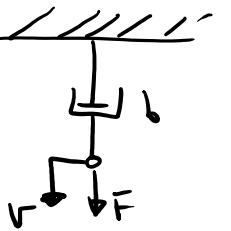
$$f = \frac{1}{R}e$$

$$\frac{e}{f} \rightarrow R : R$$

$$\left. \begin{aligned} e &= \Phi_r(f) \\ f &= \Phi_r^{-1}(e) \end{aligned} \right\} \text{non-linear}$$

Common resistances

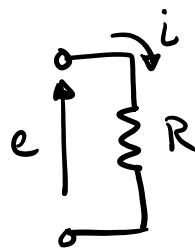
viscous damper/dashpot



$$\frac{F}{v} \rightarrow R : b$$

$$F = bv$$

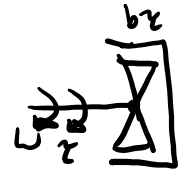
electric resistor



$$e = Ri \text{ Ohm's Law}$$

$$\frac{e}{i} \rightarrow R : R$$

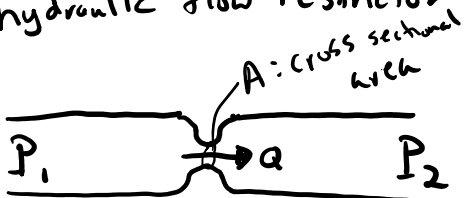
rotational viscous damper



$$\tau = b_r \omega$$

$$\frac{\tau}{\omega} \rightarrow R : b_r$$

hydraulic flow restrictor



$$\Delta P = P_1 - P_2$$

$$\Delta P = \frac{e}{2A^2} Q|Q| = \frac{e}{2A^2} \text{sgn}(Q)Q^2$$

Non-linear

$$\frac{\Delta P}{Q} \propto R$$

1 port compliance (capacitance)

- relates effort e and displacement q

$$\frac{e}{f} \rightarrow C$$

remember

$$E = \int e f dt = \int e dq$$

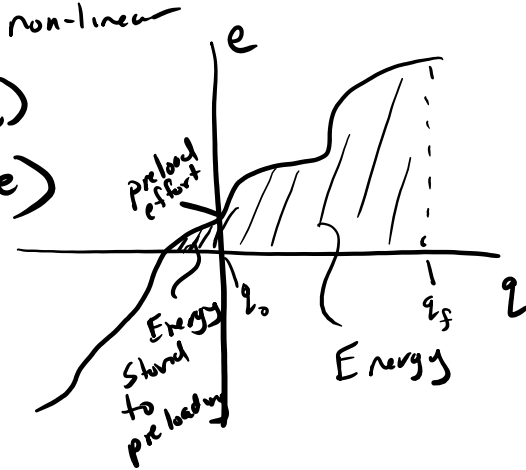
and

$$f dt = dq$$

$$q = \int f dt$$

compliance stores energy! no loss of energy: conservative, storage of potential energy

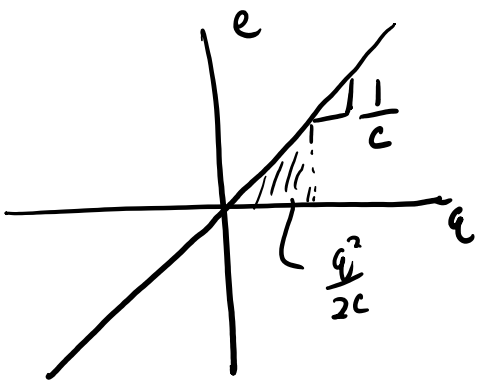
sign convention: if e is + and f is +, then power into the compliance is +.



$$e = \Phi_c^{-1}(q)$$

$$q = \Phi_c(e)$$

linear version



$$e = \frac{1}{c} q$$

$$q = c e$$

$$E = \int e dq$$

$$E = \int \frac{1}{c} q dq$$

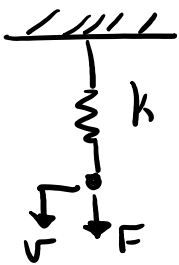
$$E = \frac{1}{2c} q^2$$

$q = \int f dt$ } need two equations
 ' ' ' and f

$$\left. \begin{aligned} q &= \int f dt \\ e &= \Phi_c^{-1}(q) \end{aligned} \right\} \begin{aligned} &\text{need two equations} \\ &\text{to relate } e \text{ and } f \\ &e = \Phi_c^{-1}(\int f dt) \end{aligned}$$

Example

Spring

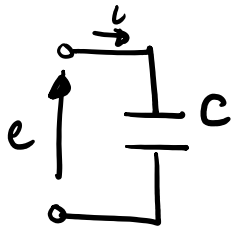


$$F = kx$$

$$x = \int v dt$$

$$\frac{F}{v} \rightarrow C : \frac{1}{k}$$

capacitor

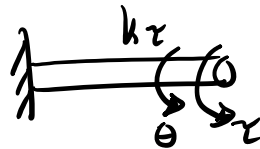


$$q = Ce$$

$$q = \int i dt$$

$$\frac{e}{i} \rightarrow C : C$$

torsion bar

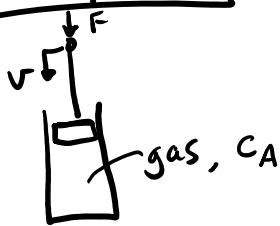


$$\tau = k_t \theta$$

$$\theta = \int \omega dt$$

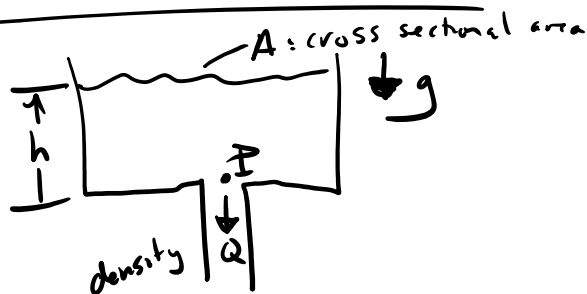
$$\frac{\tau}{\omega} \rightarrow C : \frac{1}{k_t}$$

air spring



$$\frac{F}{v} \rightarrow C : C_A$$

Small tank of fluid



$$P = \rho g h = \rho g \frac{V}{A}$$

$$V = \frac{A}{\rho g} P$$

$$q = Ce$$

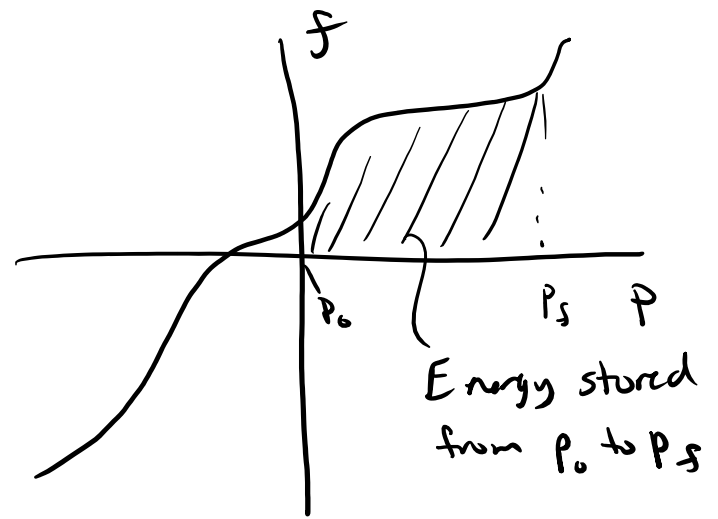
$$\frac{P}{Q} \rightarrow C : \frac{A}{\rho g}$$

Q - 03

1 port inertia

- relates flow and momentum

$\frac{e}{f} \triangle I$ $p = \Phi_I(f)$
 $f = \Phi_I^{-1}(p)$

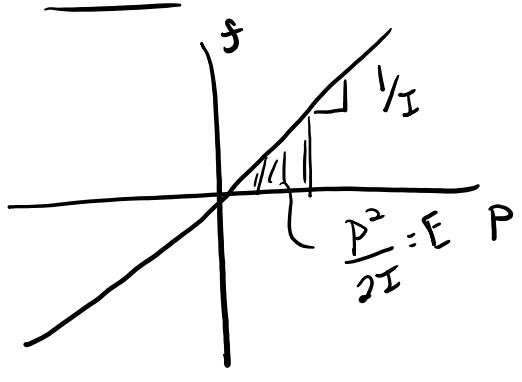


$E = \int e f dt = \int f dp$
 $e = \frac{dp}{dt} \Rightarrow e dt = dp$

+e, +f \Rightarrow + power into the
 I part, storing "kinetic energy"

$p = \int e dt$
 $f = \Phi_I^{-1}(p)$ } $f = \Phi_I^{-1}(\int e dt)$

linear



$f = \frac{1}{I} p$
 $p = I f$

$\frac{e}{f} \triangle I : I$

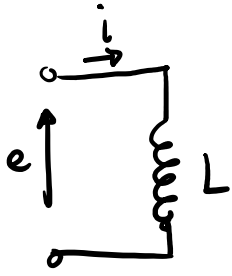
Examples

inductor (stores "kinetic" energy in magnetic field)

mass

rotating distributed

inductor (store energy in a magnetic field)



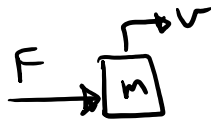
$$\frac{e}{i} \rightarrow I : L$$

flux \rightarrow

$$\lambda = Li$$

$$\lambda = \int e dt$$

mass

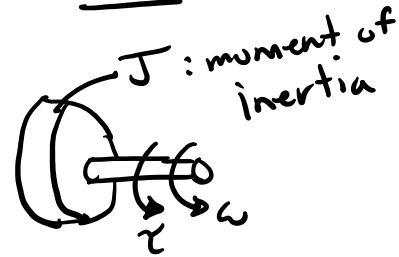


$$\frac{F}{v} \rightarrow I : m$$

$$p = mv$$

$$p = \int F dt$$

rotating distributed mass



$$h = J\omega$$

$$h = \int \tau dt$$

$$\frac{\tau}{\omega} \rightarrow I : J$$

Non-linear example: mass moving close to the speed of light

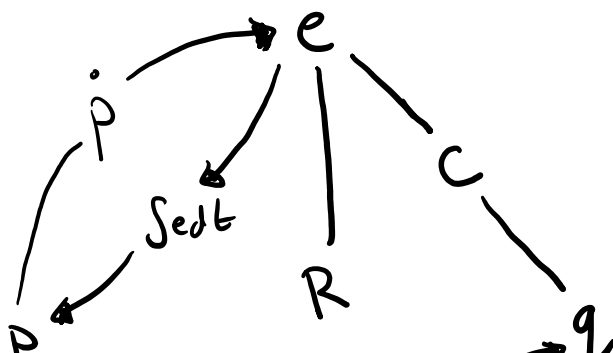
$$p \neq mv$$

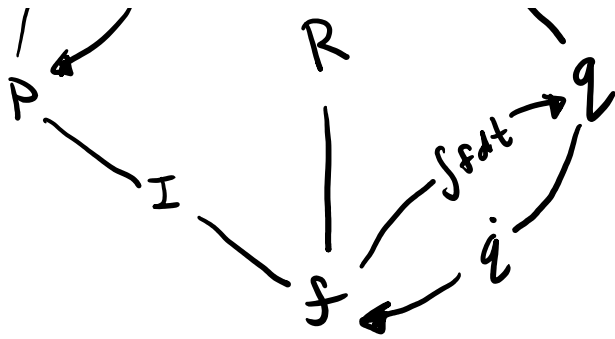
$$p = \underline{Q}_I(v) = \frac{mv}{\left(1 - \frac{v^2}{c^2}\right)^{1/2}}$$

\hookrightarrow speed of light

Relationship of the 4 parts

Tetrahedron of State





1 port effort and flow "sources"

$S_e \rightarrow$ supplies a specified effort, "draws" needed flow

$S_f \rightarrow$ supplies a specified flow, "draws" needed effort

Example: battery

$e: S_e \frac{e}{i}$ specifies a constant voltage, "draws" needed current

Example: gravity @ earth's surface

$mg: S_e \frac{mg}{v}$ specifies a constant force, "draws" needed velocity

Example: road input from lab 1

$v_{in}: S_f \frac{F}{v_{in}}$ specifies a vertical velocity, "draws" needed force

2 port components

Transformers

$$\frac{e_1}{f_1} \rightarrow \overset{m}{T} F \frac{e_2}{f_2}$$

$$e_1 = m e_2$$

$$m f_1 = f_2$$

effort γ effort

flow γ flow

no energy lost: conservative