

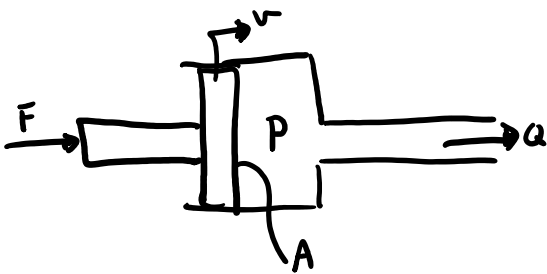
- Lab 3 due tomorrow
- Lab 4 introduced tomorrow
- HW 7 Due Monday

Transducers

Transformers and Gytrors that connect energy domains.

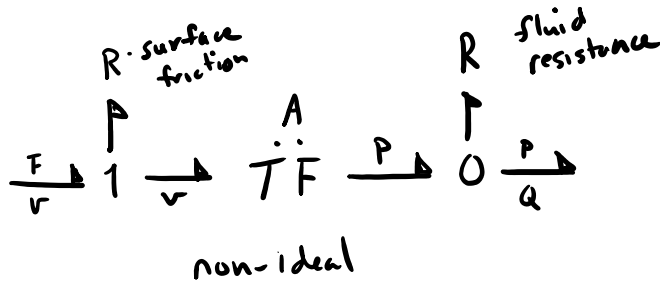
Some examples:

hydraulic ram

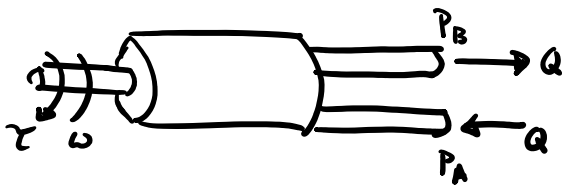


$$F = AP$$

$$Av = Q$$

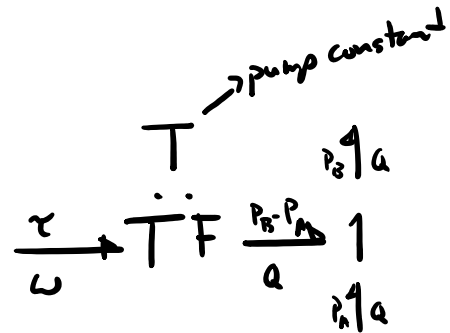


hydraulic pump (positive displacement)

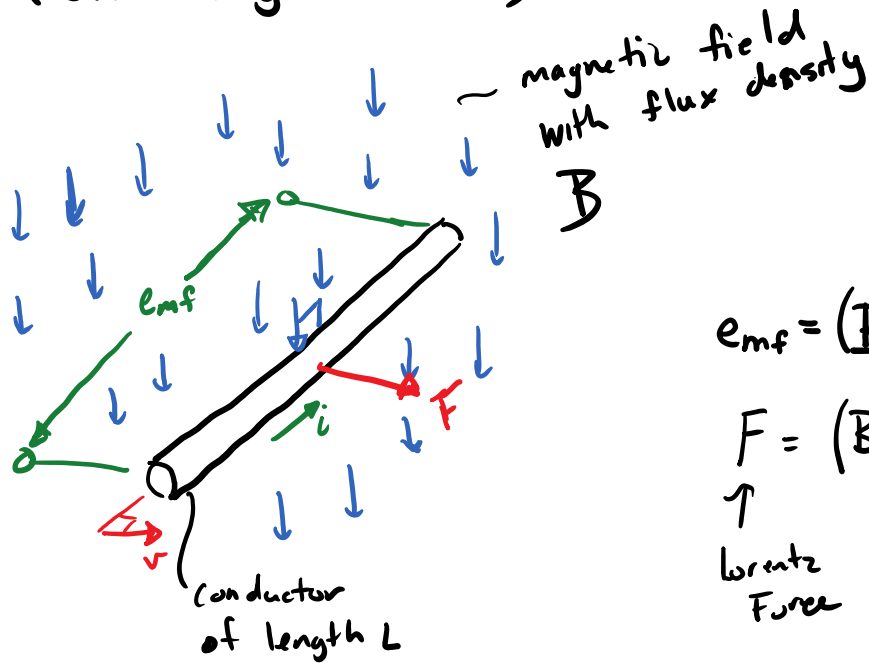


$$\tau = T(P_B - P_A)$$

$$T\omega = Q$$



Electrical \leftrightarrow Mechanical transducers are based the Lorentz force (electromagnetic force).



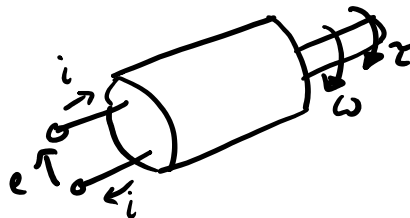
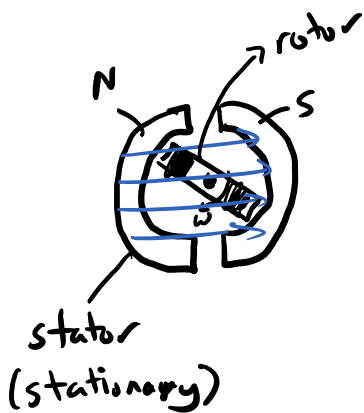
$$emf = (BL)v \quad \text{Faraday's Law}$$

$$F = (BL)i \quad \text{Lorentz's Law}$$

\uparrow
Lorentz Force

$$\frac{emf}{i} \rightarrow \overset{BL}{\dot{G}} \Upsilon \frac{F}{v}$$

A wound permanent magnet DC motor (uses Lorentz force to generate a torque)

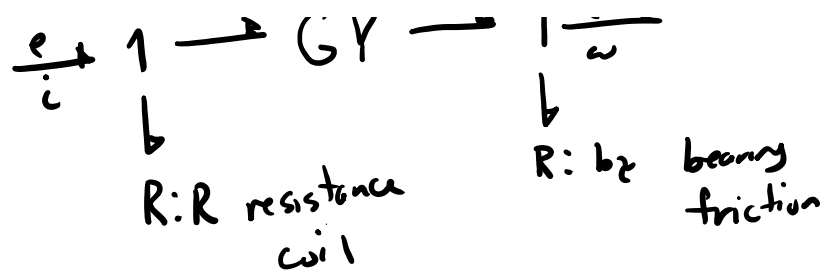


$$\frac{e}{i} \rightarrow \overset{T}{\dot{G}} \Upsilon \frac{\omega}{L} \rightarrow \text{motor constant}$$

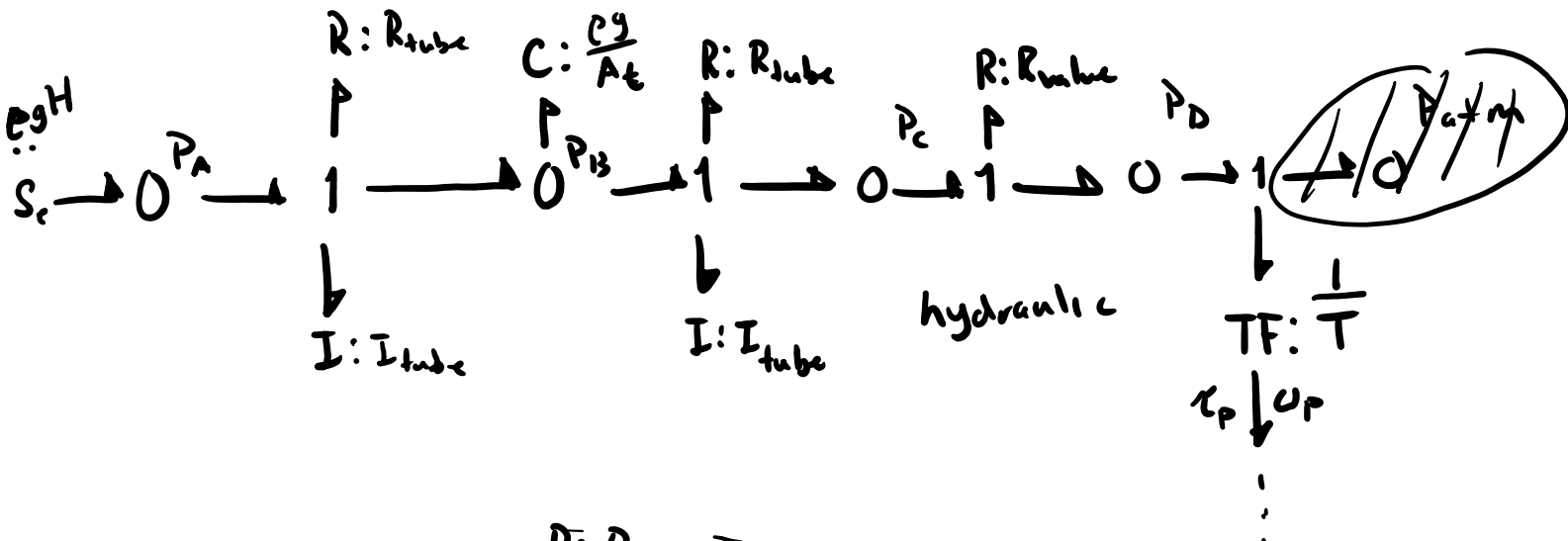
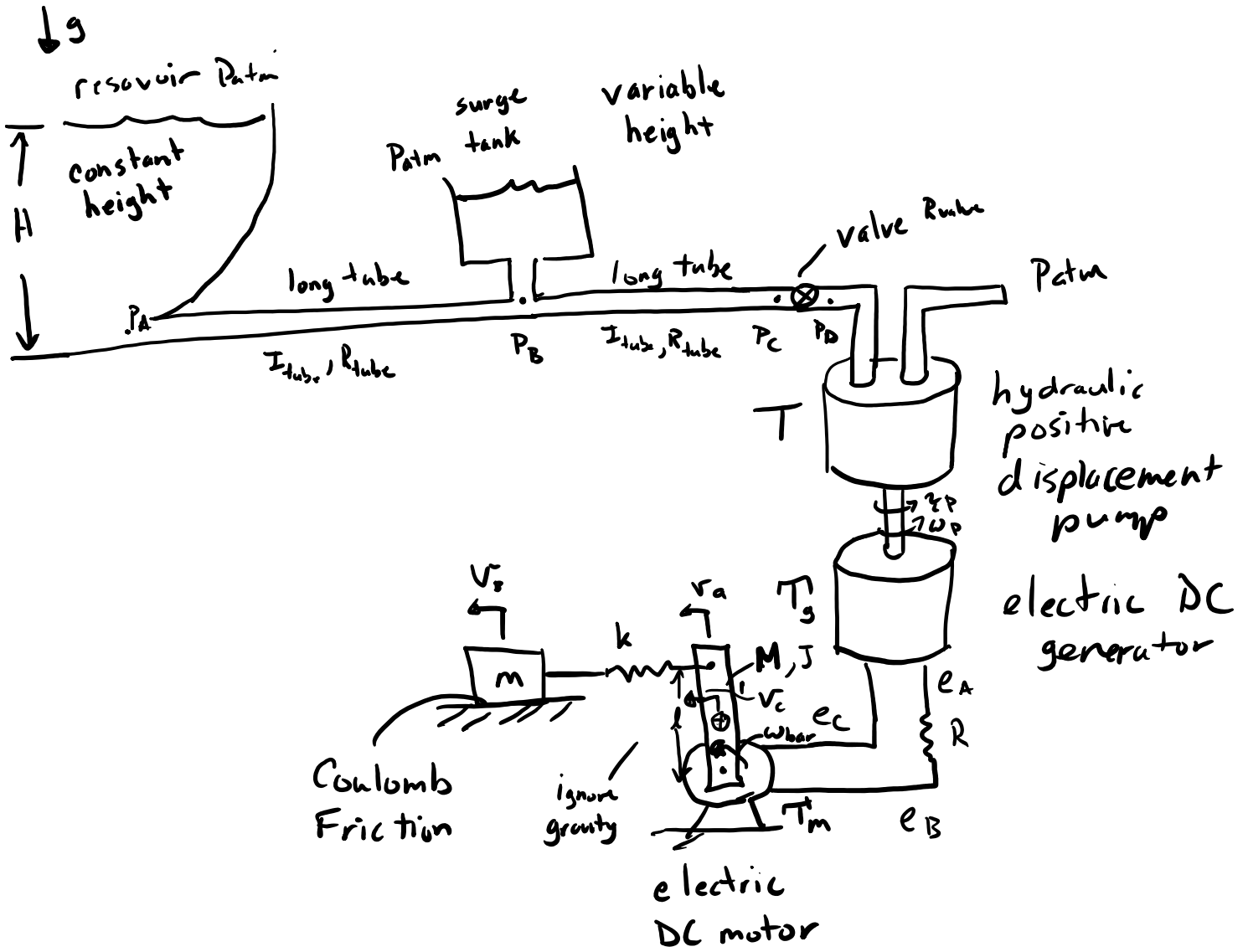
$$e = T\omega$$

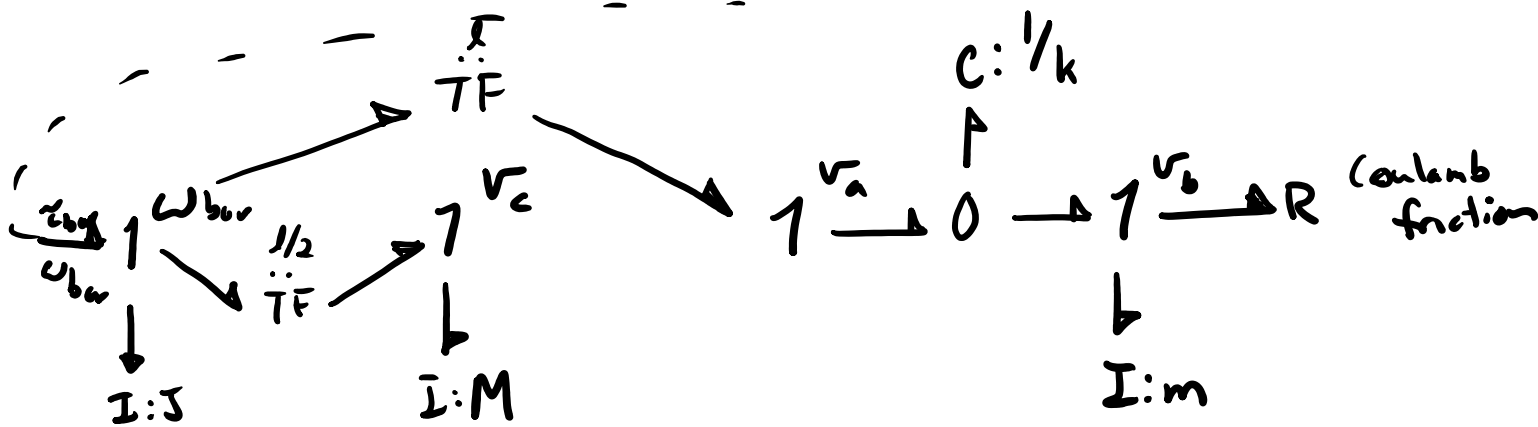
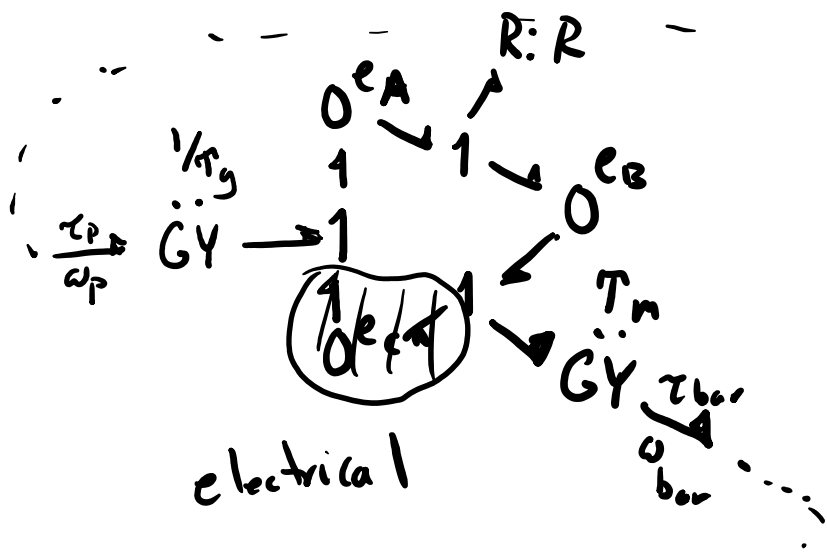
$$Ti = \omega$$

$$\frac{e}{i} \rightarrow \overset{I:L \text{ inductance}}{\uparrow} 1 \rightarrow \overset{T}{\dot{G}} \Upsilon \rightarrow \overset{I:J \text{ rotor inertia}}{\uparrow} 1 \frac{\omega}{L}$$



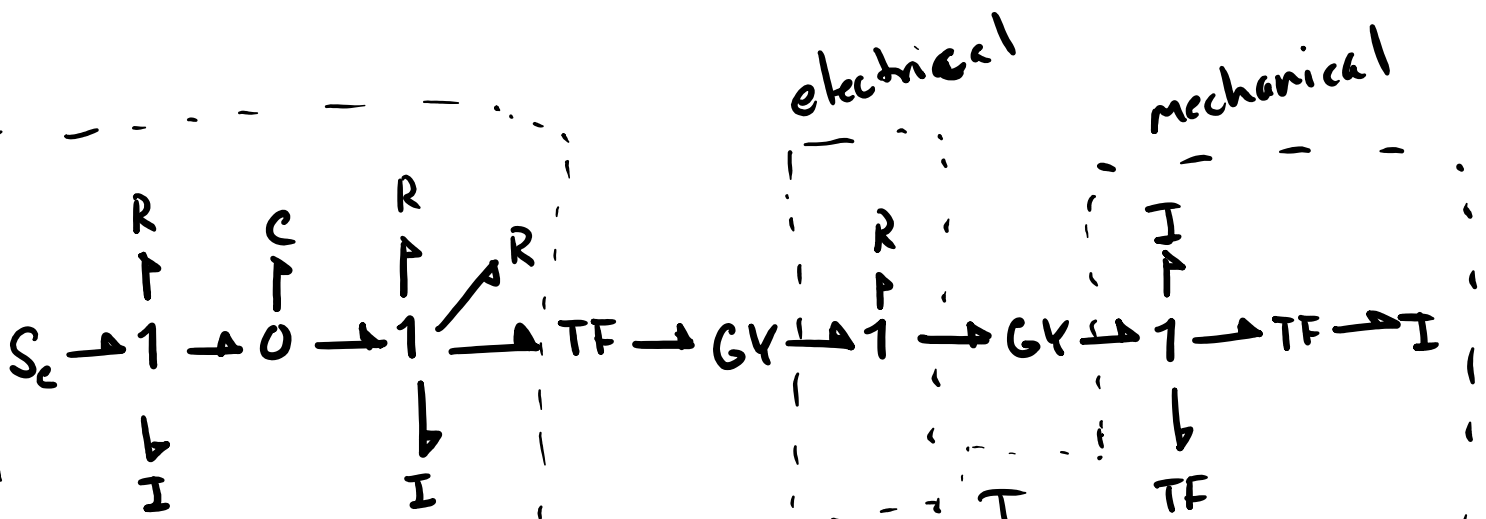
Example: Multi-Domain





$$\omega_{bar} l = v_a$$

$$\omega_{bar} l/2 = v_c$$



I

I

hydraulic

I

TF

↓

R

↑

0

C

Chapter 5: Generating System Equations From Bond Graphs

State determined systems: future depends on the present state as well as inputs to the system

Can be described by a set of non-linear explicit n first order differential equations:

$$\left. \begin{array}{l} \dot{x}_1 = f_1(x_1, x_2, \dots, x_n, u_1, \dots, u_s, t) \\ \vdots \\ \dot{x}_n = f_n(x_1, x_2, \dots, x_n, u_1, \dots, u_s, t) \end{array} \right\} \dot{\bar{x}} = \bar{f}(\bar{x}, \bar{u}, t)$$

$\bar{x} \triangleq$ state vector, $\bar{u} \triangleq$ input vector

Linear version of these equations:

$$\left. \begin{array}{l} \dot{x}_1 = a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n + b_{11}u_1 + \dots + b_{1s}u_s \\ \vdots \\ \dot{x}_n = a_{n1}x_1 + a_{n2}x_2 + \dots + a_{nn}x_n + b_{n1}u_1 + \dots + b_{ns}u_s \end{array} \right\} \dot{\bar{x}} = A\bar{x} + B\bar{u}$$

state space form

A : state matrix
 $n \times n$

B : input matrix
 $n \times s$

n : number of states
 s : number of inputs

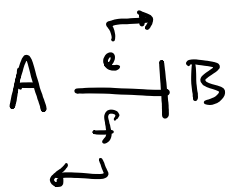
$$A = \begin{bmatrix} a_{11} & \dots & a_{1n} \\ \vdots & & \vdots \\ a_{m1} & \dots & a_{mn} \end{bmatrix}$$

$$B = \begin{bmatrix} b_{11} & \dots & b_{1s} \\ \vdots & & \vdots \\ b_{m1} & \dots & b_{ms} \end{bmatrix}$$

$$\begin{matrix} n \times 1 \\ \dot{\bar{x}} \end{matrix} = \begin{matrix} n \times n & n \times 1 \\ A & \bar{x} \end{matrix} + \begin{matrix} n \times s & s \times 1 \\ B & u \end{matrix}$$

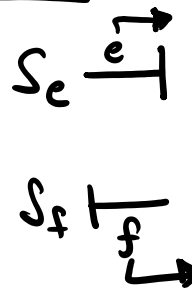
Causality Refresher

general



	A	B
input	f	e
output	e	f

Sources



resistance

$$\frac{e}{f} \text{ R:R } e = Rf$$

$$\frac{e}{f} \text{ R:R } f = \frac{1}{R}e$$

Capacitance

preferred causality

$$\frac{e}{f} \text{ C:C } e = \frac{1}{C}q = \frac{1}{C} \int f dt \text{ integral}$$

$$\frac{e}{f} \text{ C:C } f = \frac{dq}{dt} = C \frac{de}{dt} \text{ derivative}$$

Inertia

preferred causality

$$\frac{e}{f} \text{ I:I } f = \frac{1}{I}p = \frac{1}{I} \int e dt \text{ integral}$$

$$\frac{e}{f} \text{ I:I } e = \frac{dp}{dt} = I \frac{df}{dt} \text{ derivative}$$

Transformers

$$\frac{e_1}{f_1} \text{ TF } \frac{e_2}{f_2} \quad e_1 = m e_2 \quad f_2 = m f_1$$

$$\frac{e_1}{f_1} \text{ TF } \frac{e_2}{f_2} \quad e_2 = \frac{1}{m} e_1 \quad f_1 = \frac{1}{m} f_2$$

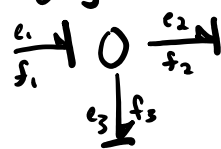
Gyrator

$$\frac{e_1}{f_1} \text{ GY } \frac{e_2}{f_2} \quad e_1 = r f_2 \quad e_2 = r f_1$$

$$\frac{e_1}{f_1} \text{ GY } \frac{e_2}{f_2} \quad f_1 = \frac{1}{r} e_1 \quad f_2 = \frac{1}{r} e_2$$

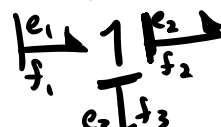
0's and 1's

only 1 effort input

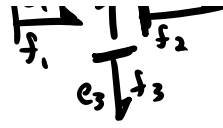


$$e_2 = e_1 \quad e_3 = e_1 \quad f_1 = f_2 + f_3$$

only 1 flow input



$$f_2 = f_1 \quad f_3 = f_1$$



$$f_3 = f_1$$
$$e_1 = e_2 + e_3$$