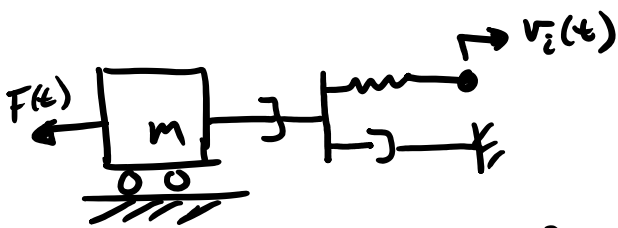


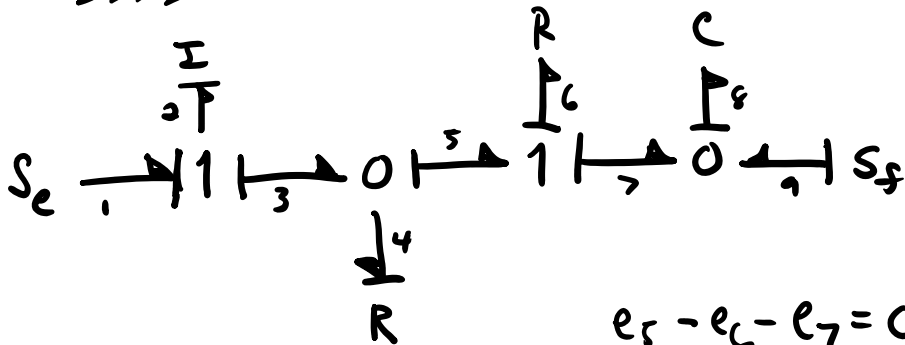
- HW 8 due tomorrow
- Lab 4 due Friday

Dealing with Algebraic Loops

Algebraic loops arise from arbitrary causality assignments to R parts.



1 arbitrary R ⇒ 1 algebraic



Inputs: S_{e1}, S_{f9}

States: P_2, q_4

$$e_5 - e_6 - e_7 = 0$$

$$e_5 = e_6 + e_7$$

$$-e_5 = -e_6 - e_7$$

$$\dot{P}_2 = e_2 = e_1 - e_3 = S_{e1} - (e_5) = S_{e1} - (e_7 + e_6) = S_{e1} - \left(\frac{q_4}{C_9} + R_6 f_6\right)$$

$$\dot{P}_2 = S_{e1} - \left(\frac{q_4}{C_9} + R_6 f_5\right) = S_{e1} - \left(\frac{q_4}{C_9} + R_6 (f_3 - f_4)\right) \quad \text{loop!}$$

$$\dot{P}_2 = S_{e1} - \left(\frac{q_4}{C_9} + R_6 \left(\frac{P_2}{I_2} - \frac{e_4}{R_4}\right)\right) = S_{e1} - \left(\frac{q_4}{C_9} + R_6 \left(\frac{P_2}{I_2} - \frac{e_5}{R_4}\right)\right)$$

$$e_5 = \frac{q_4}{C_9} + R_6 \left(\frac{P_2}{I_2} - \frac{e_5}{R_4}\right)$$

5 C_8 ... n_4)

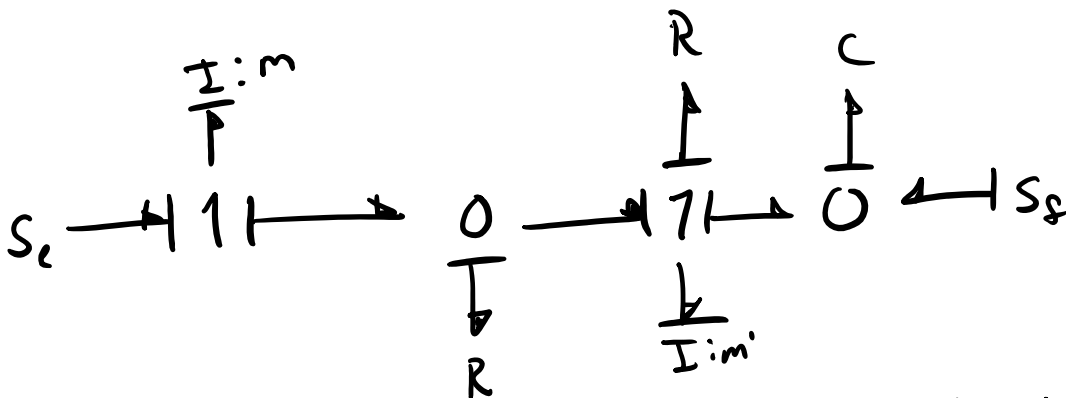
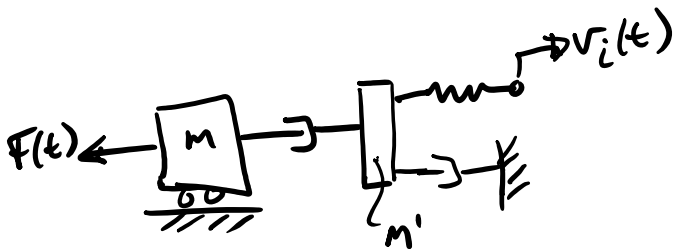
$$e_5 + \frac{R_6}{R_4} e_5 = \frac{q_8}{C_8} + \frac{R_6}{I_2} P_2$$

solve for e_5 !

$$e_5 = \frac{q_8/C_8 + \frac{R_6}{I_2} P_2}{1 + \frac{R_6}{R_4}}$$

$$\dot{P}_2 = S_{e_1} - \frac{q_8/C_8 + \frac{R_6}{I_2} P_2}{1 + R_6/R_4}$$

Way around algebraic causality



no algebraic loop, but have to track 1 more state

Derivative Causality

$\dot{\bar{x}} = f(\bar{x}, t) \Rightarrow$ explicit form: all derivatives are on the left hand side

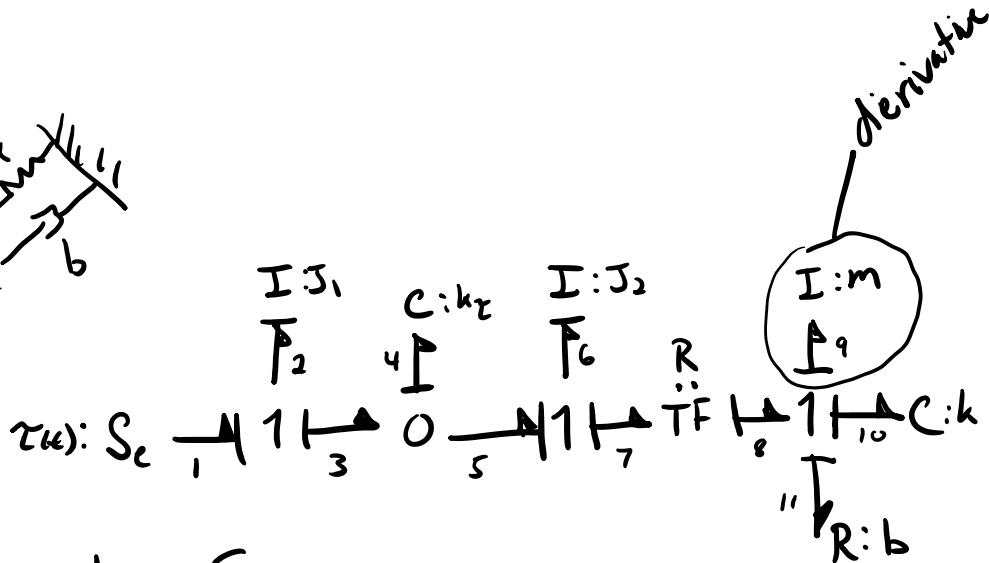
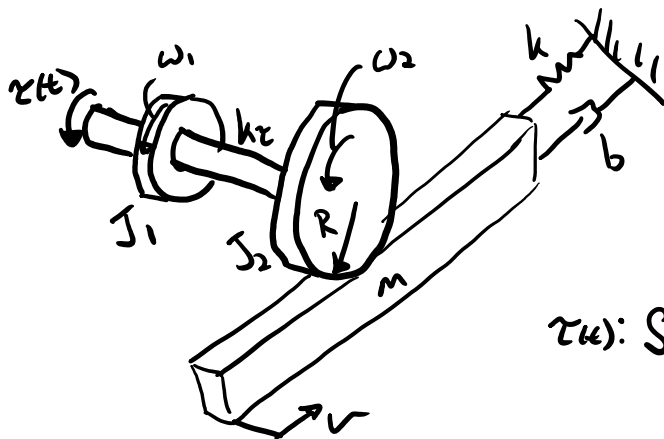
Derivative causality causes equations to be in implicit form:

$\dot{\bar{x}} = f(\dot{\bar{x}}, \bar{x}, t) \Rightarrow$ implicit: derivatives on left + right sides

Linear systems \Rightarrow turn implicit form into explicit with same algebra

Nonlinear systems \Rightarrow might not be possible to use algebra to turn into explicit form. If not possible you will have a differential algebraic equation (DAE)

Example



Inputs: S_e

States: P_2, q_4, P_6, q_{10} (not P_9 !)

States: P_2, q_4, P_C, q_{10} (not P_9 !)

$$\textcircled{1} \dot{P}_2 = e_2 = S_{e_1} - e_3 = S_{e_1} - \frac{q_4}{C_4}$$

$$\textcircled{2} \dot{q}_4 = f_4 = f_3 - f_5 = \frac{P_2}{I_2} - \frac{P_C}{I_C}$$

$$\textcircled{3} \dot{P}_C = e_6 = e_5 - e_7 = \frac{q_4}{C_4} - R e_8 = \frac{q_4}{C_4} - R(e_{10} + e_9 + e_{11})$$

$$\dot{P}_C = \frac{q_4}{C_4} - R \left(\frac{q_{10}}{C_{10}} + \dot{P}_9 + R_{11} R \frac{P_C}{I_C} \right)$$

$$I_{\text{port}} \Rightarrow P = I f$$

For each derivative causality bond write the constitutive

Law:

$$P_9 = I_9 f_9 = I_9 R \frac{P_C}{I_C}$$

take time derivative

$$\dot{P}_9 = \frac{I_9 R}{I_C} \dot{P}_C$$

$$\text{plug in for } \dot{P}_9 \Rightarrow \dot{P}_C = \frac{q_4}{C_4} - R \left(\frac{q_{10}}{C_{10}} + \frac{I_9 R}{I_C} \dot{P}_C + R_{11} R \frac{P_C}{I_C} \right)$$

solve for \dot{P}_C

$$\textcircled{3} \dot{P}_C = \frac{\frac{q_4}{C_4} - R \left(\frac{q_{10}}{C_{10}} + R_{11} R \frac{P_C}{I_C} \right)}{1 + \frac{I_9 R^2}{I_C}}$$

$$(4) \dot{q}_{10} = f_{10} = R \frac{p_c}{I_c}$$

If two or more derivative causality bonds you may see something like this:

$$\ddot{p}_c = a \dot{p}_c + b \dot{q}_{10} + \dots$$

$$\dot{q}_{10} = c \dot{p}_c + d \dot{q}_{10} + \dots$$

$$(1-a)\dot{p}_c - b\dot{q}_{10} = \dots$$

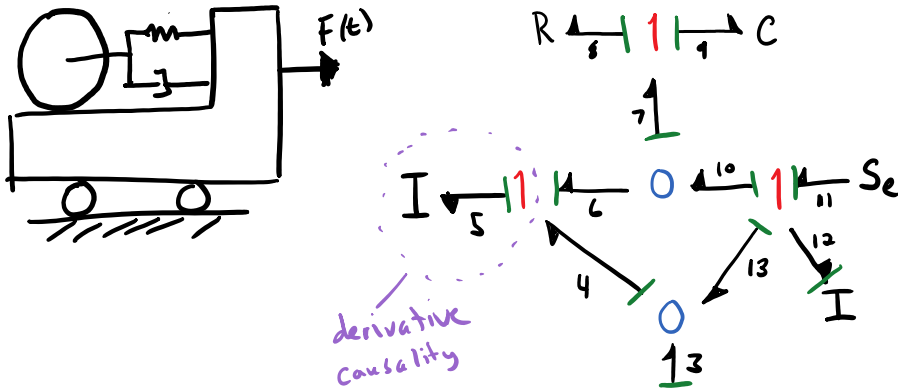
$$-c\dot{p}_c + (1-d)\dot{q}_{10} = \dots$$

$$\begin{bmatrix} 1-a & -b \\ -c & 1-d \end{bmatrix} \begin{bmatrix} \dot{p}_c \\ \dot{q}_{10} \end{bmatrix} = \begin{bmatrix} \dots \\ \dots \end{bmatrix}$$

$$A \bar{x} = \bar{b}$$

$$\bar{x} = A^{-1} \bar{b}$$

$$\text{matlab} \Rightarrow x = A \backslash b$$



Inputs: S_{e11}

States: P_1, q_1, P_{12}

Derivative causality bond:

$P_5 = I_5 f_5$ constitutive law

$P_5 = I_5 (f_{13} + f_3) = I_5 \left(\frac{P_{12}}{I_{12}} + R \frac{P_1}{I_1} \right)$

$\dot{P}_5 = \frac{I_5}{I_{12}} \dot{P}_{12} + \frac{RI_5}{I_1} \dot{P}_1$ P_1 & P_{12} are states

System equations

$\dot{P}_1 = e_1 = -e_2 = -Re_3 = -Re_4 = -R(e_5 - e_6) = -R(\dot{P}_5 - (e_6 + e_9))$

$\dot{P}_1 = -R \left[\dot{P}_5 - (R_8 f_8 + \frac{q_9}{C_9}) \right] = -R \left(\dot{P}_5 - R_8 (f_{10} - f_6) - \frac{q_9}{C_9} \right)$

$\dot{P}_1 = -R \left(\dot{P}_5 - R_8 \left(\frac{P_{12}}{I_{12}} - (f_3 + f_{13}) \right) - \frac{q_9}{C_9} \right)$

$\dot{P}_1 = -R \left[\dot{P}_5 - R_8 \left(\frac{P_{12}}{I_{12}} - \left(R \frac{P_1}{I_1} + \frac{P_{12}}{I_{12}} \right) \right) - \frac{q_9}{C_9} \right]$

$\dot{P}_1 = -R \dot{P}_5 + \frac{RR_8}{I_{12}} P_{12} - \frac{R^2 R_8}{I_1} P_1 - \frac{RR_8}{I_{12}} P_{12} + \frac{R}{C_9} q_9$

① $\dot{P}_1 = -\frac{RI_5}{I_{12}} \dot{P}_{12} - \frac{R^2 I_5}{I_1} \dot{P}_1 - \frac{R^2 R_8}{I_1} P_1 + \frac{R}{C_9} q_9$

② $\dot{q}_9 = f_9 = f_{10} - f_6 = -\frac{R}{I_1} P_1$

$$P_1 = \frac{\left(1 + \frac{I_5}{I_{12}}\right) \left(\frac{R}{C_9} q_9 + \frac{R^2 R_8}{I_1} P_1\right) - \frac{R I_5}{I_{12}} S_{e11}}{\left(1 + \frac{R^2 I_5}{I_1}\right) \left(1 + \frac{I_5}{I_{12}}\right) - \frac{R^2 I_5^2}{I_1 I_{12}}}$$

$$P_{12} = \frac{-R \frac{I_5}{I_1} \left(\frac{R}{C_9} q_9 + \frac{R^2 R_8}{I_1} P_1\right) + \left(1 + \frac{R^2 I_5}{I_1}\right) S_{e11}}{\left(1 + \frac{R^2 I_5}{I_1}\right) \left(1 + \frac{I_5}{I_{12}}\right) - \frac{R^2 I_5^2}{I_1 I_{12}}}$$