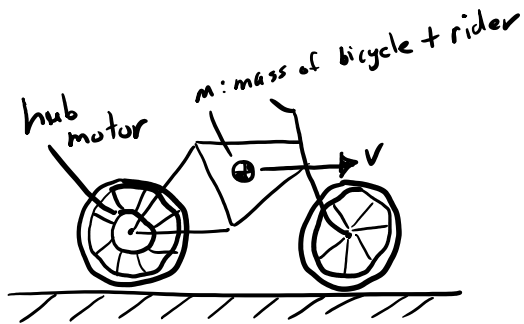


What you will do in this class?

Ebike model example



electrical power
makes motor spin
and propel bicycle
forward

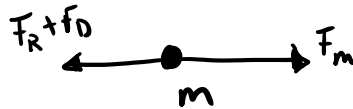
electrical + rotational + translational
3 domains

Modeling

- assumptions :
- linear motion of bicycle
 - battery has infinite energy and infinite amps can be drawn
 - motor is an ideal DC motor
 - bicycle is a particle

Translational

Free Body Diagram



F_m : propulsive force of motor

F_R : rolling resistance

F_D : air drag

F_R assumed to be constant wrt to speed, v .

$$F_R = \underbrace{mg}_{\text{weight}} \underbrace{C_R}_{\text{coefficient of rolling resistance}}$$

$0 < C_R$

F_D : blunt body traveling through air

$$F_D = \frac{1}{2} C_D \rho A v^2$$

coeff. of Drag air density frontal area speed

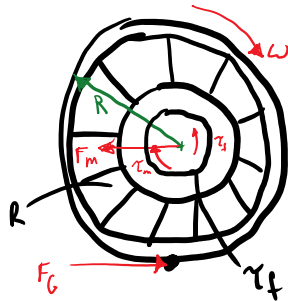
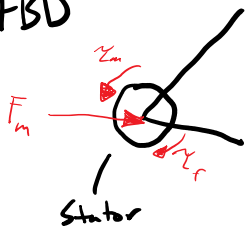
Newton's 2nd Law

$$m\dot{v} = F_m - F_R - F_D$$

$$m\dot{v} = F_m - mg C_R - \frac{1}{2} C_D \rho A v^2$$

Rotational

FBD



$$\Sigma M_{\text{avk}} = -\tau_f + \tau_m - F_G R$$

$$\Sigma F = F_m - F_G = 0$$

$$F_m = F_G$$

$$J\dot{\omega} = \tau_m - b_r \omega - F_G R$$

Electrical



$$L \frac{di}{dt} = -i R_w - V_{\text{emf}} + V$$

$$L \frac{di}{dt} = -i R_w - k_r \omega + V$$

Extra

$$\left. \begin{aligned} \tau_m &= k_r i \\ V_{\text{emf}} &= k_r \omega \end{aligned} \right\} \text{DC motor torque current relationship}$$

Final Equations

- ① $m\dot{v} = F_m - mg C_R - \frac{1}{2} \rho C_D A v^2$
- ② $\dot{x} = v$
- ③ $J\dot{\omega} = k_r i - b_r \omega - F_m R$
- ④ $\dot{\theta} = \omega$
- ⑤ $L \frac{di}{dt} = -i R_w - k_r \omega + V$

3 equations to track 5 variables x, v, θ, ω, i

Since the tire is assumed not to slip:

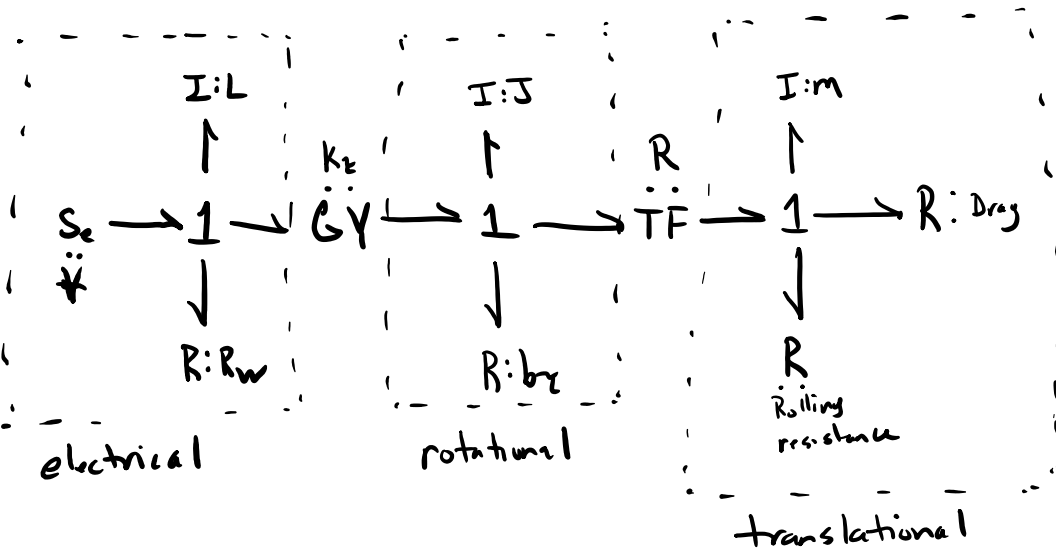
$$(6) V = \omega R$$

And F_m can be eliminated giving:

$$\begin{aligned}
 (J + mR^2) \dot{\omega} &= k_L i - b_L \omega - mg C_R R - \frac{1}{2} \rho C_D A v^2 R \\
 \dot{\theta} &= \omega \\
 L \frac{di}{dt} &= -i R_W - k_L \omega + V \\
 \dot{x} &= \omega R
 \end{aligned}$$

reduced to 4 equations

These equations can also be represented by something called a bond graph. This is what it looks like:

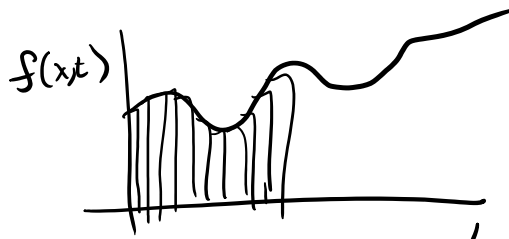


4 1st order ordinary differential equations

They tell us how x, v, θ, ω, i change with time.

Must integrate to find: $x(t), \theta(t), \omega(t), i(t)$.

$$f(x, t) = \frac{dx}{dt}$$



↳ integrate
along t by segmenting
and summing area
under the curve

In matlab `ode45()` will numerically integrate
a set of ordinary differential equations.

See `simulate_ebike.m` on the
course website for details
of how these 4 equations were
implemented in matlab.