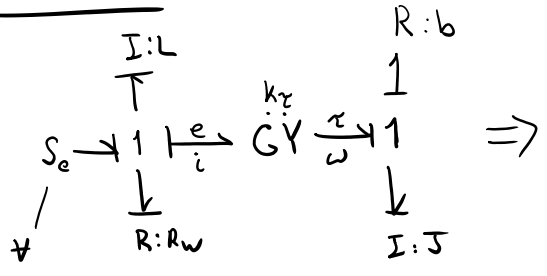
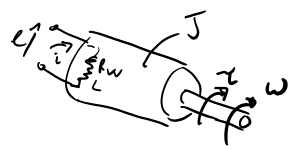


DC Motor



$\dot{q} = i$
 $\dot{i} = (-R_w i - k_T \omega + V) / L$
 $\dot{\theta} = \omega$
 $\dot{\omega} = (-b \omega + k_T i) / J$

resulting system dynamic equations
 4 first order explicit ordinary differential equation



Constant Parameter (constant w/ time)

- L : motor inductance [Henry]
- R_w : motor winding resistance [ohm]
- k_T : motor torque constant [Nm/A]
- b : rotary viscous damping coeff. [Nm·s]
- J : rotary inertia [kgm²]

State variables (time varying)

- $q(t)$: charge
 - $i(t)$: current
 - $\theta(t)$: angle
 - $\omega(t)$: angular rate
- unknowns

Input variables (time varying)

- $V(t)$: voltage specified by us

Matrix Form

$\bar{X} = \begin{bmatrix} q \\ i \\ \theta \\ \omega \end{bmatrix}$

$\bar{P} = \begin{bmatrix} L \\ R_w \\ k_T \\ b \\ J \end{bmatrix}$

$\dot{\bar{X}} = \frac{d\bar{X}}{dt} = \bar{F}(\bar{X}, \bar{P}, t)$

$\dot{\bar{X}} = \begin{bmatrix} i \\ (-R_w i - k_T \omega + V(t))/L \end{bmatrix}$

$\bar{X}(t)$ $\begin{bmatrix} \ddot{\omega} \end{bmatrix}$
state vector

$\begin{bmatrix} b \\ J \end{bmatrix}$
constants vector

$$\dot{\bar{X}} = \begin{bmatrix} 0 \\ (-R\omega i - k_T \omega + v(t))/L \\ \omega \\ (-b\omega + k_T i)/J \end{bmatrix} = \begin{bmatrix} \dot{\omega} \\ \ddot{\omega} \\ \omega \\ \dot{\omega} \end{bmatrix}$$

$$v(t) = 1$$

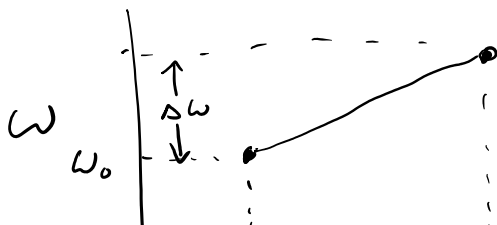
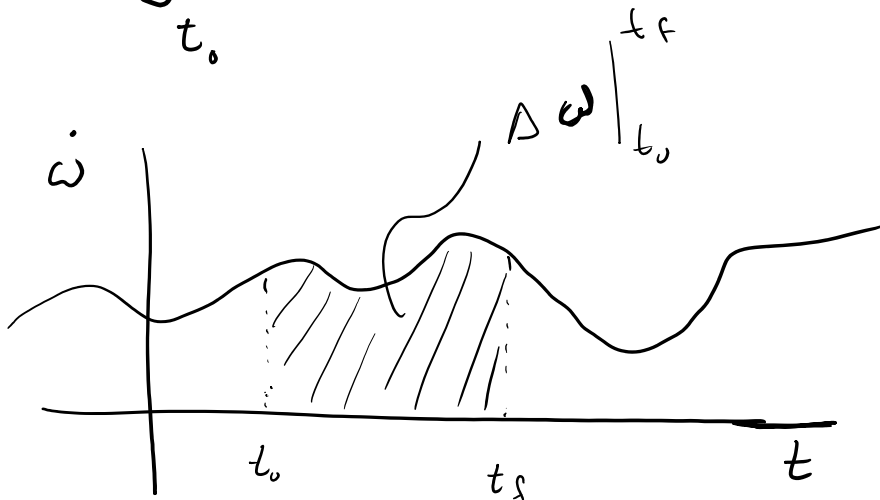
$$v(t) = \sin(\pi t)$$

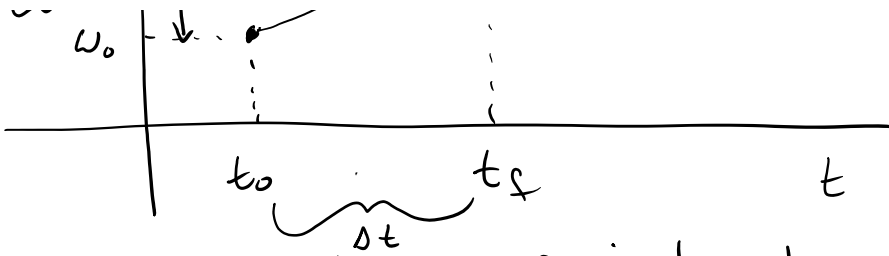
$$v(t) = \begin{cases} 0 & t < 1 \\ 1 & t \leq 1 \end{cases}$$

Integrate $\dot{\bar{X}}$ to find \bar{X} .
 \uparrow know \uparrow unknown

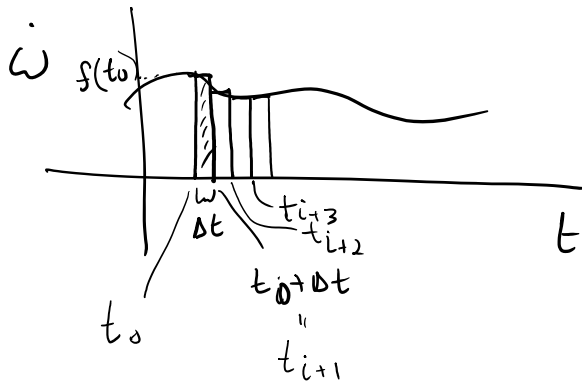
$$\bar{X} = \int_{t_0}^{t_f} \dot{\bar{X}} dt$$

$$\Delta \omega = \int_{t_0}^{t_f} \ddot{\omega} dt$$





Euler's Method of integration



$$\omega(t_0 + \Delta t) - \omega(t_0) = \dot{\omega}(t_0) \Delta t$$

$$\omega(t_0 + \Delta t) = \omega(t_0) + \Delta t \dot{\omega}(t_0)$$

Euler's
integration
method

For \bar{x} :

$$\bar{x}(t_i + \Delta t) = \bar{x}(t_i) + \Delta t \underbrace{\dot{\bar{x}}(t_i)}_{\bar{f}(\bar{x}(t_i), \bar{p}, t_i)}$$