

Chapter 5: Generating Sys. Eqns From Bond Graphs

state determined systems: future depends on present state and present input (not past)

Non-linear explicit first order ordinary differential equation

$$\begin{aligned} \dot{x}_1 &= f_1(x_1, x_2, \dots, x_n, u_1, \dots, u_s, t) \\ &\vdots \\ \dot{x}_n &= f_n(x_1, x_2, \dots, x_n, u_1, \dots, u_s, t) \end{aligned}$$

} $\dot{\bar{x}} = \bar{f}(\bar{x}, \bar{u}, t)$
matrix or vector form of the sys. eqns

$x \Rightarrow$ state variables $x(t)$
 $u \Rightarrow$ inputs $u(t)$
 t

linear version

$$\dot{x}_1 = a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n + b_{11}u_1 + \dots + b_{1s}u_s$$

\vdots

$$\dot{x}_n = a_{n1}x_1 + a_{n2}x_2 + \dots + a_{nn}x_n + b_{n1}u_1 + \dots + b_{ns}u_s$$

$$\overset{n \times 1}{\dot{\bar{x}}} = \overset{n \times n}{A} \overset{n \times 1}{\bar{x}} + \overset{n \times s}{B} \overset{s \times 1}{\bar{u}}$$

n : # states

s : # inputs

↳ matrix form "state space form"

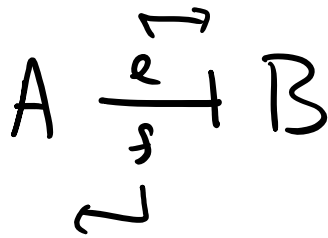
State matrix

$$A = \begin{bmatrix} a_{11} & \dots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{n1} & \dots & a_{nn} \end{bmatrix}$$

input matrix

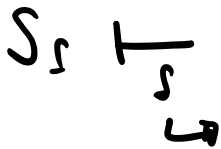
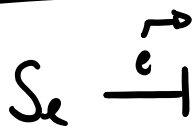
$$B = \begin{bmatrix} b_{11} & \dots & b_{1s} \\ \vdots & \ddots & \vdots \\ b_{n1} & \dots & b_{ns} \end{bmatrix}$$

Causality Refresh

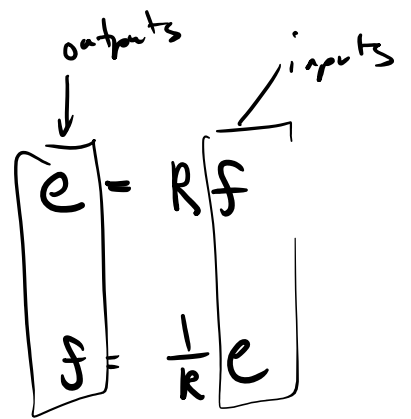


	A	B
input	f	e
output	e	f

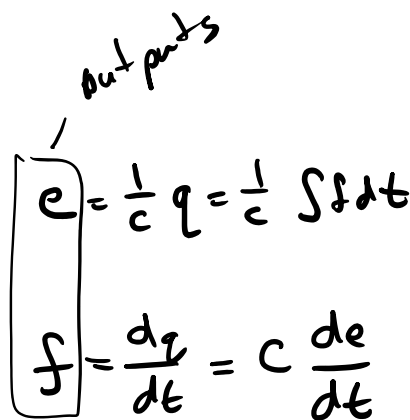
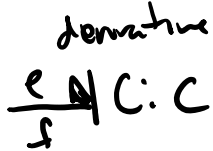
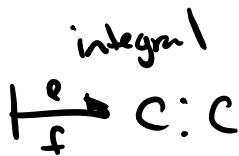
Sources



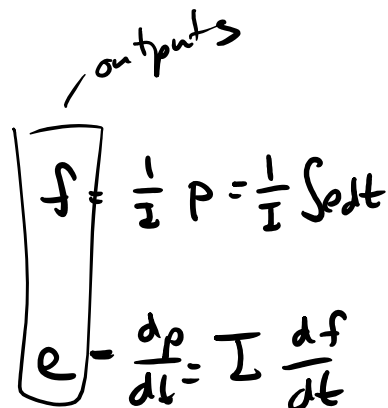
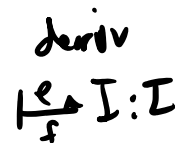
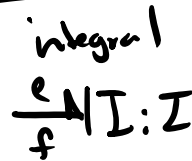
Resistor



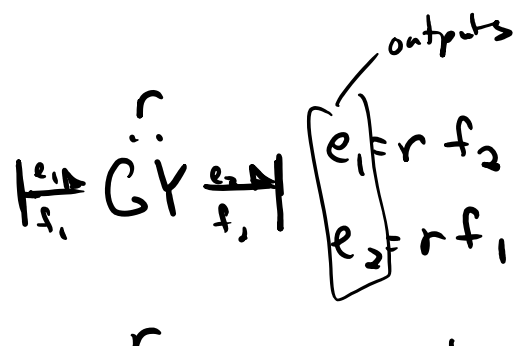
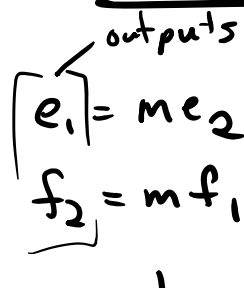
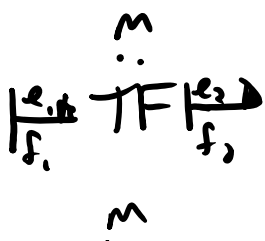
Capacitor

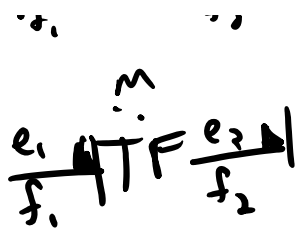


Inertion



Transformers + Gyroscopes

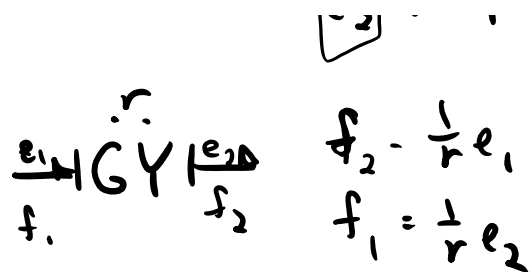




$$r_2 = m r_1$$

$$e_2 = \frac{1}{m} e_1$$

$$f_1 = \frac{1}{m} f_2$$

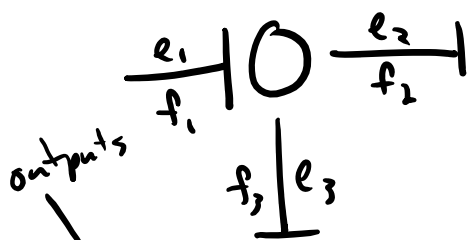


$$f_2 = \frac{1}{r} e_1$$

$$f_1 = \frac{1}{r} e_2$$

0 and 1 junctions

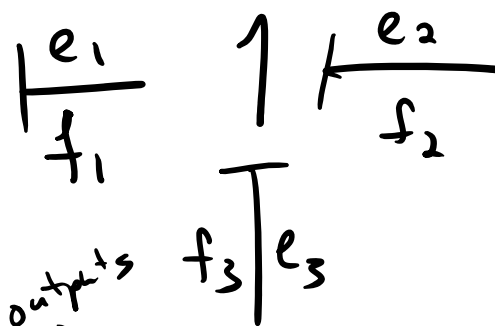
one effort input!



outputs

$$\begin{cases} e_2 = e_1 \\ e_3 = e_1 \\ f_1 = f_2 + f_3 \end{cases}$$

one flow input!



outputs

$$\begin{cases} f_2 = f_1 \\ f_3 = f_1 \\ e_1 = e_2 + e_3 \end{cases}$$

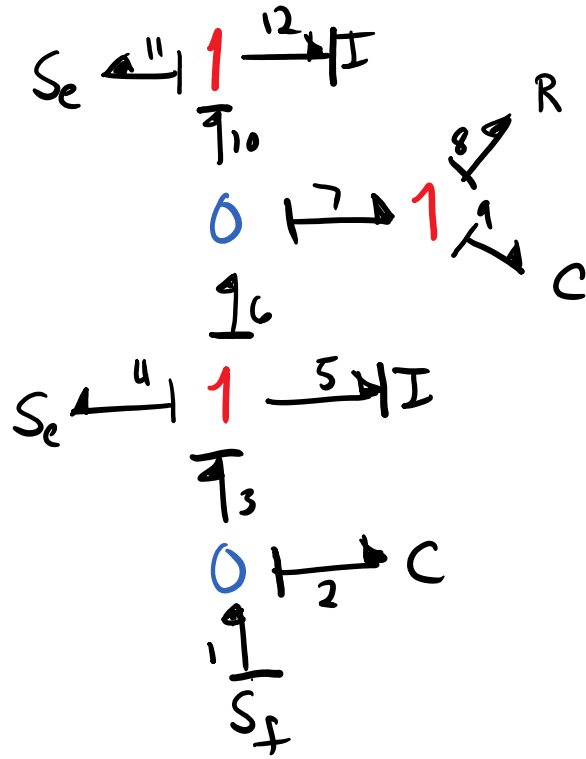
Assigning Causality to System Models

Steps

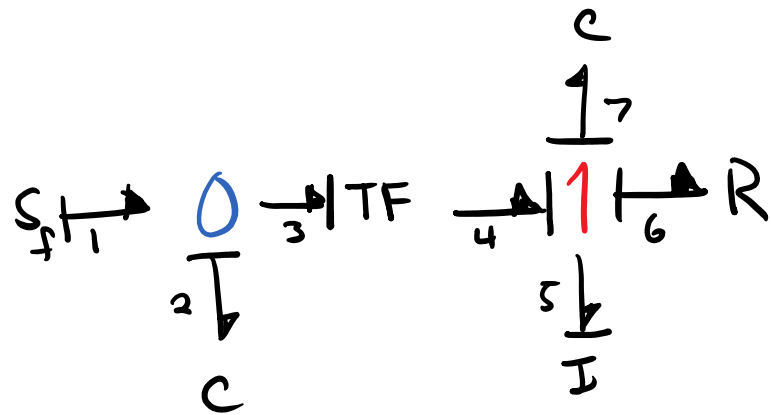
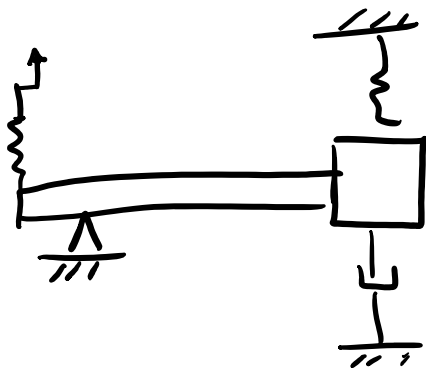
1. Number bonds
2. Choose an S_e or S_f and assign causality
Extend causality through TF, GY, O, I as far as possible.
4. Repeat 2 & 3 for all S_e and S_f
5. Select an I or C and assign the preferred integral causality.
6. Extend causality. . . .
7. Repeat 5 & 6 for all I and C
8. Common that causality is complete \rightarrow check and stop. If not move to step 9.
9. Choose any R and assign an arbitrary causality.
10. Extend causality . . .

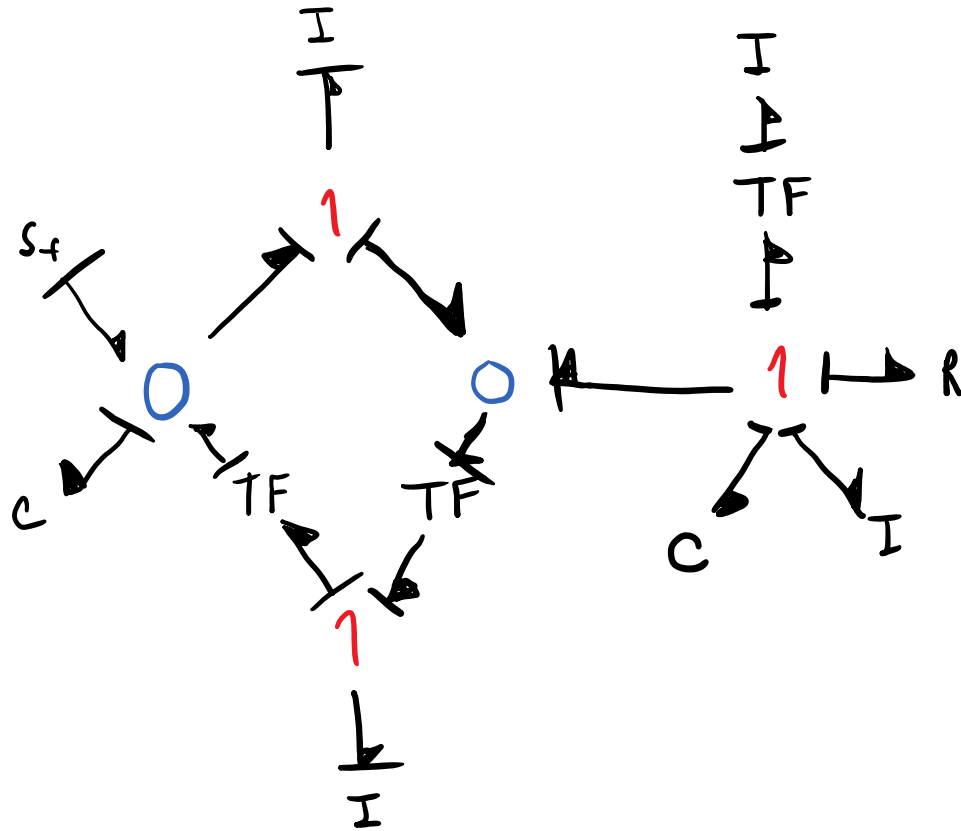
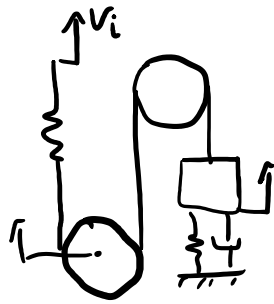
11. Repeat 9 & 10 for all R_s
12. Check for completeness, Stop if complete
13. Choose any bond without causality and assign arbitrary causality.
14. Extend . . .
15. Repeat 13 & 14 until done.

Example 1



Example 2





Steps

1. S_f, S_e
2. I, C
3. R
4. other

