

$$\omega_p = \dot{\theta}_p$$

$$\omega_w = \dot{\theta}_w$$

Torque, τ , acts between platform and wheel.

$$\Delta\theta = \theta_p + \theta_w$$

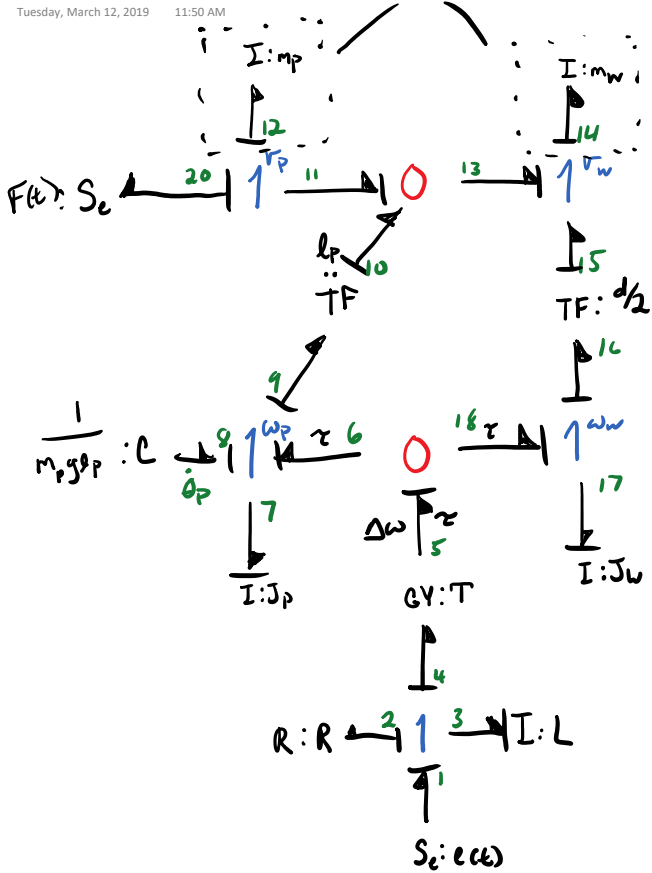
$$\Delta\omega = \omega_p + \omega_w$$

Assumptions

- Bearing friction associated with $\Delta\omega$.
- Simple DC motor with winding resistance and inductance generates $\Delta\omega$ given $e(t)$ with motor constant T .
- No-slip between wheel and ground.

platform with rigidly attached person

derivative causality



$$v_p = v_w - \omega_p l_p$$

$$v_w = \omega_w d/2$$

small angles!! no vertical velocity of person CoM

Gravity is a compliance here.

Torque caused by gravity:

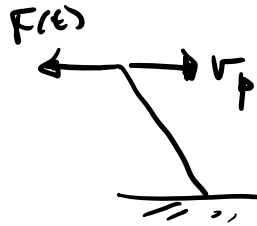
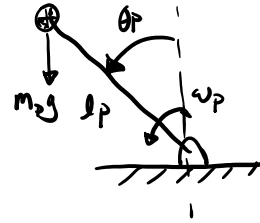
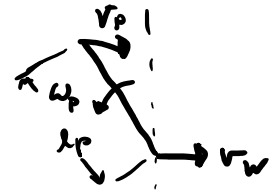
$$\tau_g = m_p g l_p \sin \theta_p$$

small angle

$$\sin \theta_p \approx \theta_p$$

$$\tau_g \approx m_p g l_p \theta_p$$

effort - displacement relationship



$$M \bar{y} = N \bar{w} + \bar{b} \bar{r}$$

$$y = \begin{bmatrix} \dot{p}_7 \\ \dot{p}_{17} \end{bmatrix} \quad \bar{w} = \begin{bmatrix} p_3 \\ q_8 \end{bmatrix}$$

$$\bar{r} = [F(t)]$$

$$\begin{bmatrix} 1 + \frac{I_{12} M^2}{I_7^2} & -\frac{I_{12} M}{I_7} \\ \frac{I_{12}}{I_7} & M_{10} M_{15} \end{bmatrix} \begin{bmatrix} \dot{p}_7 \\ \dot{p}_{17} \end{bmatrix} = \begin{bmatrix} \frac{4r_5}{I_3} \\ \frac{4r_5}{I_3} \end{bmatrix} \frac{1}{c_8} \begin{bmatrix} p_3 \\ q_8 \end{bmatrix} + \begin{bmatrix} q_{10} M_{10} \\ -16 M_{15} \end{bmatrix} [F(t)]$$

$$\bar{y} = M^{-1} (N \bar{w} + \bar{b} \bar{r})$$

$$M^{-1} = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} \quad \begin{bmatrix} \dot{p}_7 \\ \dot{p}_{17} \end{bmatrix} = \begin{bmatrix} (m_{11} + m_{12}) \frac{4r_5}{I_3} \\ (m_{21} + m_{22}) \frac{4r_5}{I_3} \end{bmatrix} \begin{bmatrix} p_3 \\ q_8 \end{bmatrix} + \begin{bmatrix} m_{11} q_{10} - m_{12} M_{15} \\ m_{21} M_{10} - m_{22} M_{15} \end{bmatrix} [F(t)]$$

Now write in $\dot{x} = Ax + Bu$ form.

$$\dot{p}_3 = e(t) - \frac{R_2}{I_3} p_3 - \frac{4r_5}{I_7} p_7 - \frac{4r_5}{I_{17}} p_{17}$$

$$q_8 = \frac{1}{I_7} p_7$$

A

B

$$\begin{bmatrix} \dot{p}_3 \\ \dot{p}_7 \\ \dot{q}_8 \\ \dot{p}_{17} \end{bmatrix} = \begin{bmatrix} -\frac{R_2}{I_3} & -\frac{4r_5}{I_7} & 0 & -\frac{4r_5}{I_{17}} \\ (m_{11} + m_{12}) \frac{4r_5}{I_3} & 0 & \frac{m_{11}}{c_8} & 0 \\ 0 & \frac{1}{I_7} & 0 & 0 \\ (m_{21} + m_{22}) \frac{4r_5}{I_3} & 0 & \frac{m_{21}}{c_8} & 0 \end{bmatrix} \begin{bmatrix} p_3 \\ p_7 \\ q_8 \\ p_{17} \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & m_{11} M_{10} - m_{12} M_{15} \\ 0 & 0 \\ 0 & m_{21} M_{10} - m_{22} M_{15} \end{bmatrix} \begin{bmatrix} e(t) \\ F(t) \end{bmatrix}$$



torque due to gravity

$$\tau_g = mgl \sin \theta$$

potential energy storage is as $\theta \rightarrow 0$